EXERCISE – III SUBJECTIVE QUESTIONS

1. The position vector of two points A and B are $6\vec{a}+2\vec{b}$ and $\vec{a}-3\vec{b}$. If a point C divides AB in the ratio 3 : 2 then show that the position vector of C is $3\vec{a}-\vec{b}$.

2. In a ΔOAB, E is the mid-point of OB and D is a point on AB such that AD : DB = 2 : 1. if OD and AE intersect at P, then determine the ratio OP : PD using vector methods.

3. If ABCD is a quadrilateral, E and F are the mid-points of AC and BD respectively, then prove that $\overline{AB}+\overline{AD}+\overline{CB}+\overline{CD}=4\overline{EF}$

4. If $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\mu(\hat{i}+\hat{j}-\hat{k})$ are two lines, then find the equation of acute angle bisector of two lines.

5. If the three successive vertices of a parallelogram have the position vectors as,
A(–3, –2, 0); B(3, –3, 1) and C(5, 0, 2). Then find
(i) Position vector of the fourth vertex D

(ii) A vector having the same direction as that of $\overrightarrow{AB}$ but magnitude equal to $\overrightarrow{AC}$

(iii) The angle between $\overrightarrow{AC}$ and $\overrightarrow{BD}$.

6. (i) If $\hat{e}_1$ and $\hat{e}_2$ are two unit vectors such that $\vec{e}_1-\vec{e}_2$ is also a unit vector, then find the angle θ between $\hat{e}_1$ and $\hat{e}_2$.

(ii) Prove that $\left(\dfrac{\vec{a}}{\vec{a}^2}-\dfrac{\vec{b}}{\vec{b}^2}\right)^2=\left(\dfrac{\vec{a}-\vec{b}}{|\vec{a}||\vec{b}|}\right)^2$

7. (i) A vector $\vec{c}$ is perpendiuclar to the vectors $2\hat{i}+3\hat{j}-\hat{k},\ \hat{i}-2\hat{j}+3\hat{k}$ and satisfies the condition $\vec{c}.(2\hat{i}-\hat{j}+\hat{k})+6=0$. Find the vector $\vec{c}$.

(ii) Given $|\vec{a}|$ = 10, $|\vec{b}|$ = 2 and $\vec{a}.\vec{b}$ = 12 then find $|\vec{a}\times\vec{b}|$.

8. Find the shortest distance between the lines :
$\vec{r}=(4\hat{i}-\hat{j})+\lambda(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(2\hat{i}+4\hat{j}-5\hat{k})$

9. (i) Given units vectors $\hat{m},\hat{n}$ and $\hat{p}$ such that $(\hat{m}\,\hat{}\,\hat{n})=\hat{p}\,\hat{}\,(\hat{m}\times\hat{n})=\alpha$, then find value of $[\,\hat{n}\hat{p}\hat{m}\,]$ in terms of α.

(ii) Let $\vec{a},\vec{b},\vec{c}$ be three units vectors and $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$. If the angle between $\vec{b}$ and $\vec{c}$ is $\dfrac{\pi}{3}$, then find the value of $|[\,\vec{a}\vec{b}\vec{c}\,]|$.

10. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k},\ \vec{b}=2\hat{i}-\hat{j}+\hat{k},\ \vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{d}=3\hat{i}-\hat{j}-2\hat{k}$, then

(i) If $\vec{a}\times(\vec{b}\times\vec{c})=p\vec{a}+q\vec{b}+r\vec{c}$, then find value of p, q are r.

(ii) Find the value of $(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c}).\vec{d}$

11. Given that $\vec{x}+\dfrac{1}{\vec{p}^2}(\vec{p}.\vec{x})\vec{p}=\vec{q}$, then show that $\vec{p}.\vec{x}=\dfrac{1}{2}(\vec{p}.\vec{q})$ and find $\vec{x}$ in terms of $\vec{p}$ and $\vec{q}$.

12. Are the following set of vectors linearly independent?

(i) $\vec{a}=\hat{i}-2\hat{j}+3\hat{k},\vec{b}=3\hat{i}-6\hat{j}+9\hat{k}$

(ii) $\vec{a}=-2\hat{i}-4\hat{k},\vec{b}=\hat{i}-2\hat{j}-\hat{k},\vec{c}=\hat{i}-4\hat{j}+3\hat{k}$

13. It is given that $\vec{x}=\dfrac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]};\vec{y}=\dfrac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]},\vec{z}=\dfrac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$, where $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors. Show that $\vec{x},\vec{y},\vec{z}$ also forms a non-coplanar system. Find the value of $\vec{x}.(\vec{a}+\vec{b})+\vec{y}.(\vec{b}+\vec{c})+\vec{z}(\vec{c}+\vec{a})$.

14. The foot of the perpendicular drawn from the origin to the plane is (4, −2, −5), then find the vector equation of plane.

15. Find the distance between the parallel planes $\vec{r}.(2\hat{i}-3\hat{j}+6\hat{k})=5$ and $\vec{r}.(6\hat{i}-9\hat{j}+18\hat{k})+20=0$.

16. If $\vec{a},\vec{b}$ are two unit vectors and θ is the angle between them, then show that

(i) $\sin\dfrac{\theta}{2} = \dfrac{1}{2}|\vec{a}-\vec{b}|$

(ii) $\cos\dfrac{\theta}{2} = \dfrac{1}{2}|\vec{a}+\vec{b}|$

17. In a tetrahedron, if two pairs of opposite edges are perpendicular, then show that the third pair of opposite edges is also perpendicular and in this case the sum of the squares of two opposite edges is the same for each pair. Also show that the segment joining the mid points of opposite edges bisect one another.

18. (i) Let $\vec{A}=2\hat{i}+\hat{k},\ \vec{B}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{C}=4\hat{i}-3\hat{j}+7\hat{k}$. Determine a vector $\vec{R}$ satisfying $\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$ and $\vec{R}.\vec{A}=0$

(ii) Find vector $\vec{v}$ which is coplanar with the vectors $\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$ and is orthogonal to the vector $-2\hat{i}+\hat{j}+\hat{k}$. It is given that the projection of $\vec{v}$ along the vector $\hat{i}-\hat{j}+\hat{k}$ is equal to $6\sqrt{3}$.

19. In triangle ABC using vector method show that the distance between the circumcentre and the orthocentre is $R\sqrt{1-8\cos A\cos B\cos C}$, where R is the circumradius of the triangle ABC.

20. Find the equation of line of intersection of the planes $\vec{r}.(3\hat{i}-\hat{j}+\hat{k})=1$ and $\vec{r}.(\hat{i}+4\hat{j}-2\hat{k})=2$.

21. (a) Use vectors to prove that the acute angle between the plane faces of a regular tetrahedron is arc cos (1/3).

(b) Use vectors to find the circum-radius and in-radius of a regular tetrahedron in terms of the length k of each edge.

22. Find the point R in which the line AB cuts the plane CDE, where position vectors of points A, B, C, D, E are respectively $\vec{a}=\hat{i}+2\hat{j}+\hat{k},\ \vec{b}=2\hat{i}+\hat{j}+2\hat{k},\ \vec{c}=-4\hat{i}+4\hat{k},\ \vec{d}=2\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{e}=4\hat{i}+\hat{j}+2\hat{k}$.

23. Show that the circumcentre of the tetrahedron OABC is given by $\dfrac{\vec{a}^2(\vec{b}\times\vec{c})+\vec{b}^2(\vec{c}\times\vec{a})+\vec{c}^2(\vec{a}\times b)}{2[\vec{a}\,\vec{b}\,\vec{c}]}$, where $\vec{a},\vec{b}\ \&\ \vec{c}$ are the position vectors of the points A, B, C respectively relative to the origin 'O'.

24. Examine for coplanarity of the following sets of points

(i) $4\hat{i}+8\hat{j}+12\hat{k},\ \ 2\hat{i}+4\hat{j}+6\hat{k},\ \ 3\hat{i}+5\hat{j}+4\hat{k},\ \ 5\hat{i}+8\hat{j}+5\hat{k}.$

(ii) $3\vec{a}+2\vec{b}-5\vec{c},\ \ 3\vec{a}+8\vec{b}+5\vec{c},\ \ -3\vec{a}+2\vec{b}+\vec{c},\ \ \vec{a}+4\vec{b}-3\vec{c}.$

25. The position vectors of the angular points of a tetrahedron are $A(3\hat{i}-2\hat{j}+\hat{k}), B(3\hat{i}+\hat{j}+5\hat{k})$, $C(4\hat{i}+\hat{k})$ and $D(\hat{i})$. Then find the acute angle between the lateral faces ADC and the base ABC.

26. The three vectors $\vec{a}=4\hat{i}-2\hat{j}+\hat{k}$; $\vec{b}=2\hat{i}-\hat{j}-\hat{k}$ and $\vec{c}=2\hat{i}+\hat{k}$ are all drawn from the point with position vector $\hat{i}-\hat{j}$. Find the equation of the plane containing their end point.

27. Line L_1 is parallel to vector $\vec{\alpha}=3\hat{i}+2\hat{j}+4\hat{k}$ and passes through a point A(7, 6, 2) and line L_2 is parallel to a vector $\vec{\beta}=2\hat{i}+\hat{j}+3\hat{k}$ and passes through a point B(5, 3, 4). Now a line L_3 parallel to a vector $\vec{r}=2\hat{i}-2\hat{j}-\hat{k}$ intersects the lines L_1 and L_2 at points C and D respectively, then find $|\overline{CD}|$.